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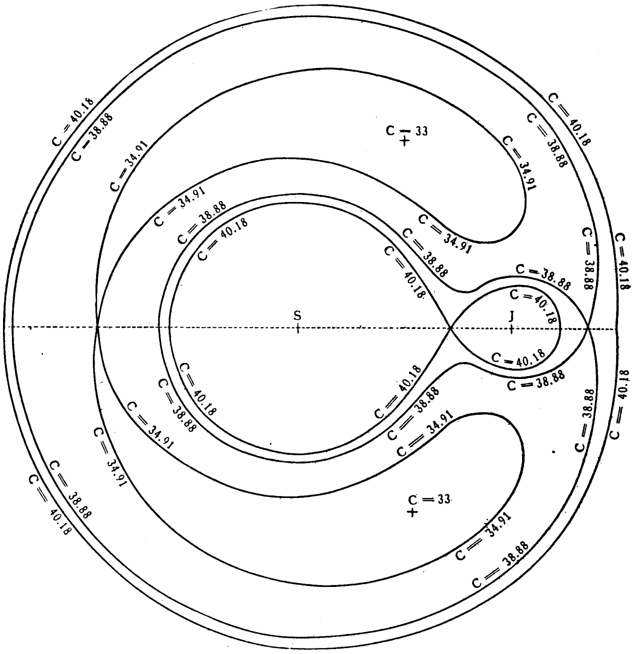

Curves of Zero Velocity (Darwin)

This diagram illustrates the hour-glass shaped space through which the particle may move and drop down nearer the sun or planet, till it becomes captured by one of the larger bodies.

PUBLICATIONS
OF THE
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THE CAPTURE THEORY OF SATELLITES.[1]

BY T. J. J. SEE.

1. THE CAPTURE THEORY OF SATELLITES, WITH APPLICATION TO THE PLANETS OF THE SOLAR SYSTEM, INCLUDING THE CASE OF OUR OWN MOON.

2. THE CAUSE OF THE OUTSTANDING SECULAR ACCELERATION IN THE MEAN MOTION OF THE MOON AND OF THE RECENTLY DISCOVERED INDICATION OF A SECULAR ACCELERATION IN THE MEAN MOTION OF THE SUN.

In the paper presented to this Society January 30, 1909, grounds were adduced to show that the planets and satellites of the solar system have not been detached from the central bodies which now govern their motions, by acceleration of rotation, as supposed by LAPLACE, but that all these masses had been captured or added from without, and have since had their orbits reduced in size and rounded up under the secular action of the nebular resisting medium formerly pervading our planetary system. In proof of this view a table was calculated to show the times of rotation of the Sun and planets when these globes were imagined expanded to fill the orbits of the bodies revolving about them, according to the mechanical principle of the conservation of areas, in the form of a criterion proposed by BABINET in 1861. This argument that the planets and satellites have been captured, and not detached, as has been generally supposed since the time of LAPLACE, is very satisfactory; and to many minds was, no doubt, quite convincing. At the time the last paper was presented the author was ill in the hospital, and it had therefore to be read by the Secretary; but even before the author's recent illness, which delayed the progress of this work several months, he was at work on a rigorous mathematical proof of the method by which the satellites had been captured, as the most effective means of reinforcing and finally clinching the argument respecting the mode of formation of the solar system. We shall now give a brief outline of this new proof, referring the reader for a fuller statement to the papers which have been communicated to the *Astronomische Nachrichten* (No. 4341-42).

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[1] Abstract of report presented to the meeting of the Directors of the Astronomical Society of the Pacific, June 25, 1909.

(1) In the year 1836 the celebrated German mathematician JACOBI communicated to the Paris Academy of Sciences an integral of the differential equations of motion for the restricted problem of three bodies; the system being made up of a sun attended by a planet revolving about it in a circle, and a particle of insensible mass. JACOBI remarks that the integral may be applied to a body such as the terrestrial Moon.

(2) In 1877 Dr. G. W. HILL developed and greatly perfected the theory of JACOBI's integral, and applied it to the lunar theory in a series of celebrated papers. HILL's work has since been the basis of the profound researches of POINCARÉ, DARWIN, and others on *Periodic Orbits* and related topics in celestial mechanics.

(3) Dr. HILL showed that in the restricted problem of three bodies, implied in JACOBI's integral, there is a partition of the whole space into three parts,—one about each of the large bodies, the sun and planet, and a larger domain enclosing both bodies,—within which the power of control over the particle is vested in the two bodies individually and collectively, respectively. The closed surface about the Earth includes the orbit of the Moon, and the orbits of the other satellites in like manner are within the closed surfaces about their several planets; and Dr. HILL remarks that this arrangement is necessary to secure stability.[1] If a satellite is once within this region, with the surface of zero velocity closed about it, it cannot escape, but will always remain attached to the planet, and its radius vector will have a superior limit. How the Moon and other satellites came within these closed regions

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[1] HILL's Collected Mathematical Works, Vol. 1, p. 330.

Dr. Hill did not inquire; and subsequent investigators appear to have supposed that as these bodies cannot now escape from their planets, so also they cannot have come in from a remote distance, but must have originated where they now are. This is the view put forth by Moulton in his discussion of Professor W. H. Pickering's suggestion that *Phœbe* had been captured by *Saturn*[1]; but such reasoning is easily shown to be erroneous by the following considerations:—

(4) Jacobi's integral, as originally given by him, is based on the differential equations for unrestricted motion in *empty space,* and no account is taken of the additional terms which must be added to the differential equations of the motion of the sun, planet, and particle, when the motion is very slightly conditioned by the introduction of a nebular resisting medium, such as existed in the early history of our system, and is now observed to be widely diffused throughout nature. Jacobi's original integral, therefore, requires the addition of a secular term to represent the actual movement of a sun, planet, and particle; and the complete expression for any particle whose co-ordinates are x_i, y_i, z_i, becomes—

$$x_i^2 + y_i^2 + \frac{2(1 - \mu)}{\sqrt{(x_i - x_1)^2 + y_i^2 + z_i^2}} +$$

$$\frac{2\mu}{\sqrt{(x_i - x_2)^2 + y_i^2 + z_i^2}} = C_i + a_i t_i \ldots . (1).$$

The secular term $a_i t_i$ makes the constant C_i increase with the time.

(5) Now the surfaces of zero relative velocity, which define the closed spaces about the planets, have larger values of C the nearer we approach to the sun or planet. This is easily seen in the accompanying plate from Darwin's celebrated memoir on *Periodic Orbits.*[2] When the particle or satellite revolves against resistance therefore, the second member of (1) increases, and there is a secular shrinkage of the surface of zero relative velocity. *Accordingly the particle drops down nearer and nearer these centers, and the surface finally becomes*

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[1] *Astrophysical Journal,* October, 1905, p. 178.

[2] *Acta Mathematica,* Vol. XXI.

closed, leaving it no longer free to move about both bodies in the hour-glass shaped space, as formerly, but restricted to the sphere of influence controlled by the sun or planet individually, as the case may be. The particle which once revolved about both the sun and planet can no longer do so, but becomes an inferior planet (satellite of the sun) or a satellite of the planet.

(6) This is how all the satellites of the solar system were captured. At first they moved principally under the attraction of the Sun, and could pass from the Sun's to the planet's domain, through the neck of the hour-glass shaped space connecting the two spheres of influence. When the neck is narrow, DARWIN says that a particle which passes from the Sun's to the planet's control may revolve about it hundreds of times before quitting the planet's sphere to return again to the Sun's control. And if resistance is meanwhile encountered, so that the neck of the surface of zero velocity becomes closed, it is clear that the particle never will quit the sphere of the planet's control, but will abide there permanently as a satellite.

(7) Thus it incontestably follows that the satellites of *Jupiter, Saturn,* and other planets formerly moved about the Sun, and since they were captured have had their orbits reduced in size and rounded up under the secular action of the resisting medium formerly pervading our solar system. *Satellites may cross over the line S J before coming completely under the planet's control, in which case they will move retrograde.* In such cases the neck connecting the two spaces is extremely narrow. But as the neck usually is not so narrow as to produce crossing satellites, most of them naturally move direct, in accordance with observation. *This is the reason also why the planets have direct rotations on their axes. The planets have in no case been inverted, as some have recently supposed, in order to account for the retrograde motion of the satellites of Jupiter and Saturn.*

(8) In the case of the terrestrial Moon it is shown that the Earth simply captured one of the twenty-seven million such planets which went to form the Sun's immense mass. The Moon came to us from the depths of space, and never was a

part of the Earth, as has long been supposed. Professor Sir G. H. DARWIN's celebrated work of 1879 is shown to be based on chance coincidences, and not actual physical history. All the details of the lunar terrestrial system are known to accord with the theory that the Moon is a captured planet. (*a*) *It it shown by rigorous calculation from the theory of probability that the chances are infinity to one that the Moon was captured like the other satellites.* (*b*) *It is likewise shown that the probability is infinity to one that the Earth could not have rotated with sufficient rapidity to detach the Moon.* As the theoretical possibility of the capture of the Moon is beyond doubt, it is therefore certain that it actually occurred.

(9) The Earth never rotated much, if any, faster than at present, and is a much better timekeeper than has been supposed heretofore. It never had a high degree of oblateness, and changes in rotation have had little if any effect on the events contemplated in geological history.

(10) As the Moon came to us from celestial space, it was, when first captured, at something like double its present distance; but has since gradually neared the Earth. The orbits of the other satellites have been reduced in size and made more circular by the secular effects of resistance, and of course the same thing has occurred in the case of our satellite. Therefore it is excessively improbable that the movement which has brought the Moon nearer and nearer the Earth has entirely ceased. It must still be going on, owing to resistance and to the secular increase in the mass of the Earth and Moon, due to cosmical dust in the form of meteorites. This continued slight secular approach of the Moon to the Earth is the true physical cause of the outstanding inequality in the secular acceleration of the Moon's mean motion. According to DELAUNAY, 6″.11 is the amount of the acceleration which can be attributed to the decrease in the eccentricity of the Earth's orbit, as explained by LAPLACE in 1787. The most ancient solar eclipses indicate an observed secular acceleration of about 12″ per century;[1] while the more modern eclipses make it from

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[1] Since this was written, I have shown that all the ancient eclipses indicate a secular acceleration of about 8″. An important series of papers bearing on the subject, by NEWCOMB, COWELL, and FOTHERINGHAM, will be found in the *Monthly Notices*, 1904-1909.

8″ to 10″. Even if the total observed amount of the secular acceleration be only 8″.4, as found by NEWCOMB, there is an outstanding difference of 2″.3. The true physical cause of this outstanding difference, whether it be 2″.3 or 6″, as indicated by the most ancient eclipses, is to be found in the origin of the Moon by capture and in its subsequent secular approach to the Earth. Thus the new theory of the capture of satellites will aid us greatly in solving certain problems of the lunar theory.

By the course of reasoning outlined at the January meeting, and in these brief remarks, we have proved the following results:—

(*a*) By the application of BABINET's criterion that the planets could never have been detached from the Sun, and the satellites could never have been detached from their planets by rotation, as LAPLACE supposed. If they could not have been detached from the bodies which now govern their motions, it naturally and necessarily follows that they must have been captured or added from without.

(*b*) By the mathematical methods which are founded on JACOBI's integral and worked out with care by HILL, POINCARÉ, DARWIN, and others, that the satellites in moving about the Sun might easily pass under the control of the planets and revolve about them as temporary satellites. If these temporary satellites suffer sensible resistance or disturbance in the motion about the planets, they will drop down nearer these centers of attraction and become permanent satellites.

(*c*) As the planets and satellites could not have been detached by rotation, but could easily have been captured, it follows that the latter process of cosmogony did really occur; so that the planets were captured by the Sun, and the satellites captured by their several planets. The Moon, too, was captured by the Earth, and has ever since gradually approached our planet; this is the cause of the outstanding inequality in the secular acceleration of the Moon's mean motion. As the Earth never rotated much more rapidly than at present, it is a much better timekeeper than we have heretofore supposed. Views very similar to these, on the secular effects of a resisting medium upon the motions of the planets, were long ago put forth by the celebrated Swiss mathematician EULER, and we are only now beginning to return to them after the lapse of more than a century and a half. The results here briefly outlined appear to be proved beyond doubt, and they afford us an impressive illustration of how cautious we must be in adopting theories merely because they have been handed down by tradition. They also afford us a good idea of how much yet remains to be done in all the physical sciences. With the new methods now introduced, doubt and uncertainty may be very largely replaced by confidence and certainty in all matters relating to cosmical evolution. Henceforth the laws of cosmical evolution will be capable of definite and exact investigations, and cosmogony will become a true physical science.

U. S. NAVAL OBSERVATORY,
MARE ISLAND, CALIFORNIA, June 11, 1909.

PLANETARY PHENOMENA FOR SEPTEMBER AND OCTOBER, 1909.

BY MALCOLM MCNEILL.

PHASES OF THE MOON, PACIFIC TIME.

Last Quarter..	Sept. 6,	$11^{h}\,44^{m}$ A.M.	Last Quarter..	Oct. 5,	$10^{h}\,44^{m}$ P.M.
New Moon...	" 14,	7 9 A.M.	New Moon...	" 14,	12 13 A.M.
First Quarter..	" 22,	10 31 A.M.	First Quarter..	" 21,	11 4 P.M.
Full Moon....	" 29,	5 5 A.M.	Full Moon....	" 28,	2 7 P.M.

The autumnal equinox, the time when the Sun crosses the equator from north to south, occurs September 23d, 9 A. M., Pacific time.

Mercury is an evening star until October 12th. It then passes inferior conjunction, becoming a morning star, and will continue as morning star until December 2d. It reaches greatest east elongation on the morning of September 17th. Its distance from the Sun is then 26° 34′, but it is so much farther south than the Sun that it will remain above the horizon not quite an hour after sunset. This interval is about the same throughout the first half of September. It is hardly long enough to permit naked eye observations except in un-